QMM 7

Probability

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Events

The event is a subcollection of the sample space, as in $E = \{T\}$ for a coin flip that gives 'tails' or $E = \{5, 6\}$ for a die roll above 4.

Counting principles

Fundamental theorem

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Combinations C

The number of combinations of n elements taken r at a time is
$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

Exercises (Larson 2009)

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How many different samples of 10 units can you extract from a population of 200?

$\binom{200}{10} = \frac{200!}{190!10!}$

Discrete random variables

A **random variable** is a function that assigns a numerical value to each of the outcomes in a sample space.

For instance, in the sample space $S = \{HH, HT, TH, TT\}$, the outcomes could be assigned the numbers 2, 1, and 0, the number of ways an event depending on the number of heads in the outcome.

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If the set of values taken on by the random variable is finite, then the random variable is **discrete**.

The number of times a specific value of x occurs is the **frequency** of x and is denoted by $n(x)$.

Probability distribution

The probability of a random variable x is $P(x) = \frac{n(x)}{n(S)}$

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The **probability distribution** of the random variable is the collection of probabilities corresponding to its values.

If the range of a discrete random variable consists of m different values $\{x_1, x_2, x_3, ..., x_m\}$, then the sum of its probabilities is 1.

$$P(x_1) + P(x_2) + P(x_3) + ... + P(x_m) = 1$$

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$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$
$$= \{3, 2, 2, 1, 2, 1, 1, 0\}$$

Find the **frequency and probability distributions** of the random variable.

Random variable x	0	1	2	3
Frequency	1	3	3	1
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Expected value

Mean of the random variable

If the range of a discrete random variable consists of m different values $\{x1, x2, x3, ..., x_m\}$, then the mean, or expected value, of the random variable is

$$E(x) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + ... + x_m P(x_m)$$

Example

Random variable x	0	1	2	3
Frequency	1	3	3	1
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
Expected value	$(0)(\frac{1}{8}) +$	$(1)(\frac{3}{8}) +$	$(2)(\frac{3}{8}) +$	$(3)(\frac{1}{8})$
$= 1.5$				

Variance and standard deviation

Consider a random variable whose range is $x_1, x_2, x_3, \ldots, x_m$ with a mean of μ.

Variance of the random variable

$$V(x) = (x_1 - \mu)^2 P(x_1) + (x_2 - \mu)^2 P(x_2) + \ldots + (x_m - \mu)^2 P(x_m)$$

Standard deviation "sigma" of the random variable

$$\sigma = \sqrt{V(x)}$$

Exercise

Assume a voting population where 80% of people never donate to political parties, 15% donate \$10 and 5% donate \$100.

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2. Find the **expected value** of the distribution.

$E(x) = 0 \cdot P(x = 0) + 10 \cdot P(x = 10) + 100 \cdot P(x = 100)$
$E(x) = 0 + .15 * 10 + .05 * 100 = 6.5$

Homework

Read Evans *et al.* 2012, ch. 3–4* for next week
and enjoy the rest of your day.

* http://www.ck12.org/book/CK-12-Advanced-Probability-and-Statistics-Concepts/