Quantitative and Mathematical Methods
Euro-American Campus · Sciences Po · Reims

Functions

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Level 1 Groups

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Functions: What do we mean by $y = f(x)$?

Functions: Basic definitions

- **Definition:**

  The real-valued function $f : x \to y \quad x, y \in \mathbb{R}$ is a rule that assigns *one* real number $y \in \mathbb{R}^1$ to each real number $x \in \mathbb{R}^1$.

  $\mathbb{R}^1$ denotes the set of all real numbers from $-\infty$ to $+\infty$ on the real number line.

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- **Domain/Range:**

  Given $f : x \in X \to y \in Y$, the set X denotes the *domain* of the function and the set Y denotes its *range* (co-domain).

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- **Euclidean space:**

  A function with n variables exists in the n-dimensional Euclidean space $\mathbb{R}^n$ where each n-axis goes from $-\infty$ to $+\infty$.

Functions: Set notation

- **Let X be a set:**

  $x \in X$: the element x belongs to the set X.
  $x \notin X$: the element x does not belong to the set X.
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- **Important sets:**

  $\mathbb{N}$: natural numbers
  $\mathbb{R}$: real numbers
  $\mathbb{Z}$: integers
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- **Interval notation:**

  $[a, b] = \{x : x \in \mathbb{R} \text{ and } a \leq x \leq b$: closed interval
  $(a, b) = \{x : x \in \mathbb{R} \text{ and } a < x < b$: open interval

Functions: Relation to sets

- **Let X and Y be sets:**

  $X = Y$: the sets X and Y are equal.
  $X \subset Y$: X is a subset of Y.
  $X \cap Y = \{x : x \in X \text{ and } x \in Y\}$: intersection
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- **Cartesian product:**

  The Cartesian product $X \times Y$ of two sets X and Y is the set of all ordered pairs (x, y) with $x, y \in \mathbb{R}$.

Ex. Let $X = \{1, 2\}$ and $Y = \{2, 4\}$.
Then $X \times Y = (1, 2), (1, 4), (2, 2), (2, 4)$.

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- **In more general terms:**

  Let X and Y be sets. The relation R from X to Y is a subset of $X \times Y$ and is written xRy if $(x, y) \in R$.

Functions: Terminology

- **Functions as mappings of sets:**

  The basic point of a function is to provide a 'rule of correspondence' between two sets: the element $x \in X$ is the **input** of the function, which produces the **output** $y \in Y$.

Functions: Terminology

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  Using the function $f : x \rightarrow y$, we might choose to express the relationship between two terms x and y as a *dependence* of y upon x. We might later say that x **predicts** y.

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- **Back to domains:**

  Let X and Y be sets. A function $f : X \to Y$ is a relation from its **domain** X to its **codomain** Y. The set $Y = f(X)$ can be written as $\{f(x) : x \in X\}$ and is the **image** (or **range**) of f.

Functions: Examples

Functions: Example

- Model of market commodity:

  Demand function: $D(x) = p$ where p is the price at which each unit of commodity x sells.
  Supply function: $S(x) = p$ where p is the price at which units of x are effectively sold.
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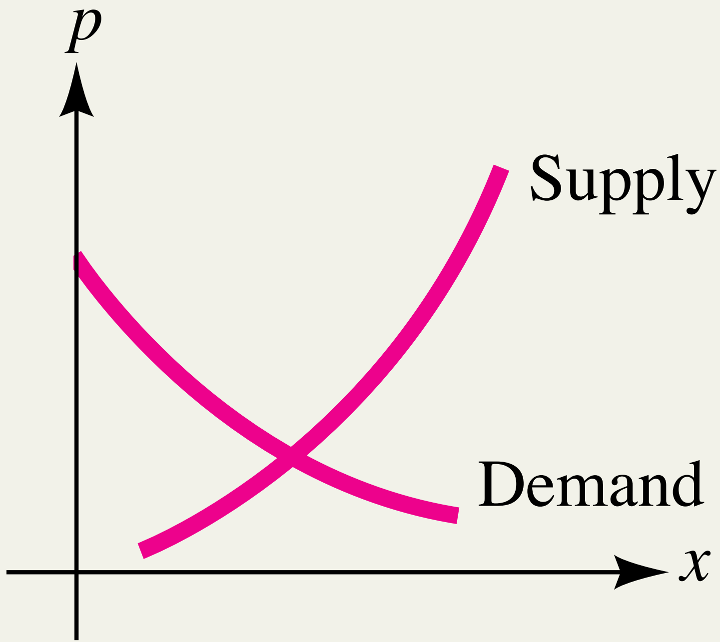
  How do these functions behave in economic view?

- Model of market profit:
  Revenue function: $R(x) = x \cdot p(x)$, which stands for (number of units sold) $\times$ (price per unit).
  Cost function: $C(x)$
  Profit function:

$$P(x) = R(x) - C(x) = x \cdot p(x) - C(x)$$



p
Supply
Demand
x

Functions: Applied example 1

- **Assume the following commodity pricing situation:**

$$p(x) = -.27x + 51 C(x) \qquad = 2.23x^2 + 3.5x + 85$$

Find $R(x)$ and $P(x)$.

Functions: Applied example 1

- **Assume the following commodity pricing situation:**

$$p(x) = -.27x + 51 C(x) \qquad = 2.23x^2 + 3.5x + 85$$

Find $R(x)$ and $P(x)$.

- **Solution 1:**

$$R(x) = x \cdot p(x) = -.27x^2 + 51x$$
$$P(x) = R(x) - C(x) = (-.27x^2 + 51x) - (2.23x^2 + 3.5x + 85)$$
$$\Rightarrow P(x) = -2.5x^2 + 47.5x - 85$$

Functions: Applied example 2

- **Following up on Example 1:**

$$P(x) = -2.5x^2 + 47.5x - 85 \text{ dollars}$$

At what values of x is production profitable?

Functions: Applied example 2

- **Following up on Example 1:**

$$P(x) = -2.5x^2 + 47.5x - 85 \text{ dollars}$$

At what values of x is production profitable?

- **Solution 2:**

$$\begin{aligned} P(x) &= -2.5x^2 + 47.5x - 85 \\ &= -2.5(x^2 - 19x + 34) \\ &= -2.5(x - 2)(x - 17) \end{aligned}$$

Production is profitable if $P(x) > 0$.
$P(x) > 0$ only if $(x - 2)(x - 17)$ is negative, i.e. when $x - 2 > 0$ and $x - 17 < 0$.
Production is profitable if $2 < x < 17$.

Functions: Applied example 3

- **Suppose the following:**
  Fixed cost function:

$$C(q) = q^3 - 30q^2 + 500q + 200$$

  Compute the production cost of 10 units.

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  Compute the production cost of 10 units.

- **Solution 3:**
  $C(10) = (10)^3 - 30(10)^2 + 500(10) + 200 = 3,200.$

Functions: Applied example 4

- Assume the following fixed cost function:

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$$C(q) = q^3 - 30q^2 + 500q + 200$$

$C(10) = 3,200$. Compute the production cost of the 10th unit.

- **Solution 4:**

$$C(10) = 3,200$$
$$C(9) = (9)^3 - 30(9)^2 + 500(9) + 200 = 2,999$$

Marginal cost of 10th unit: $C(10) - C(9) = 201$.