Quantitative and Mathematical Methods
Euro-American Campus · Sciences Po · Reims

Introduction

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Level 1 Groups

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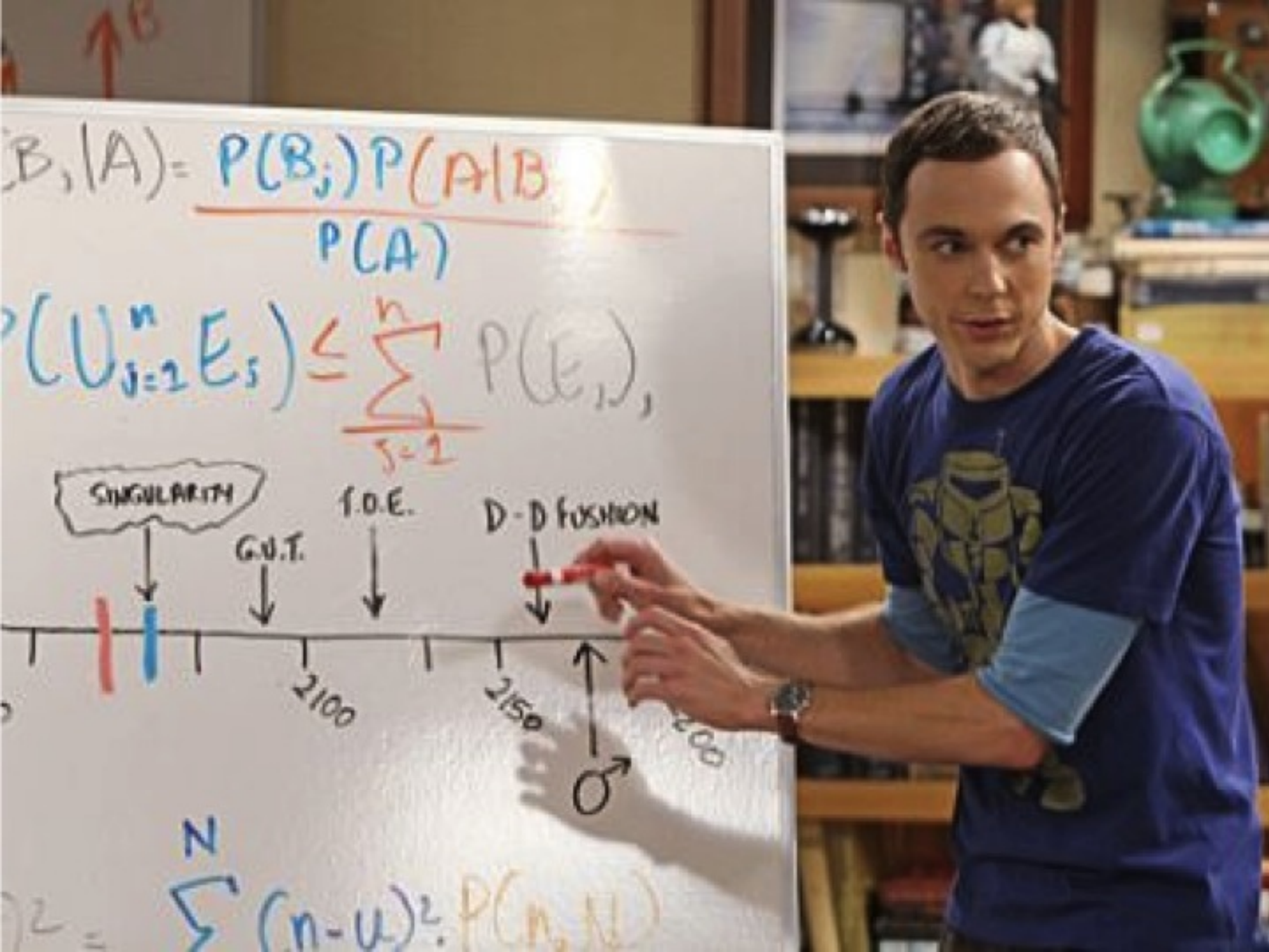
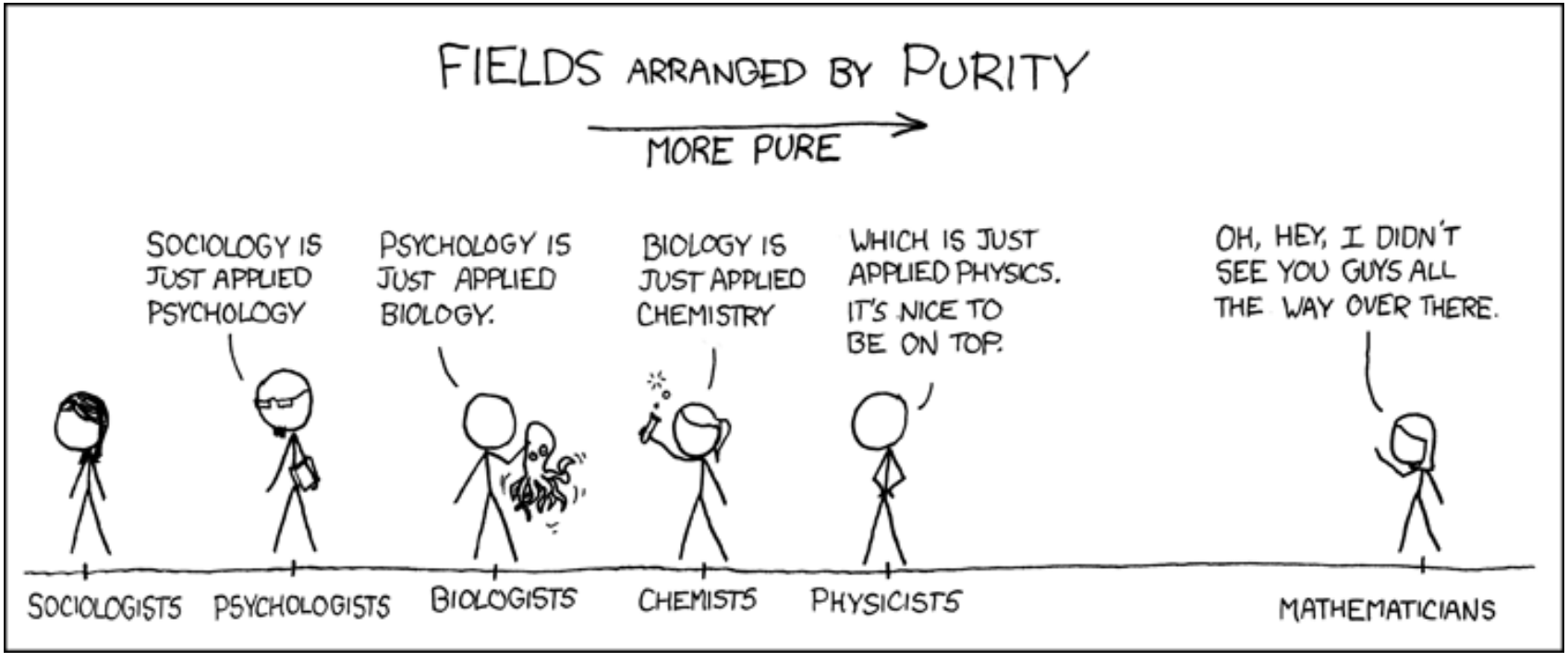
Applied probability theory


$P(B_j|A) = \frac{P(B_j)P(A|B_j)}{P(A)}$
$P(\bigcup_{j=2}^{n} E_j) \leq \sum_{j=2}^{n} P(E_j)$,
SINGULARITY
G.U.T.
T.O.E.
D-D FUSION
2100
2150
$\sum_{n=1}^{N} (n-u)^2 \cdot P(n,u)$

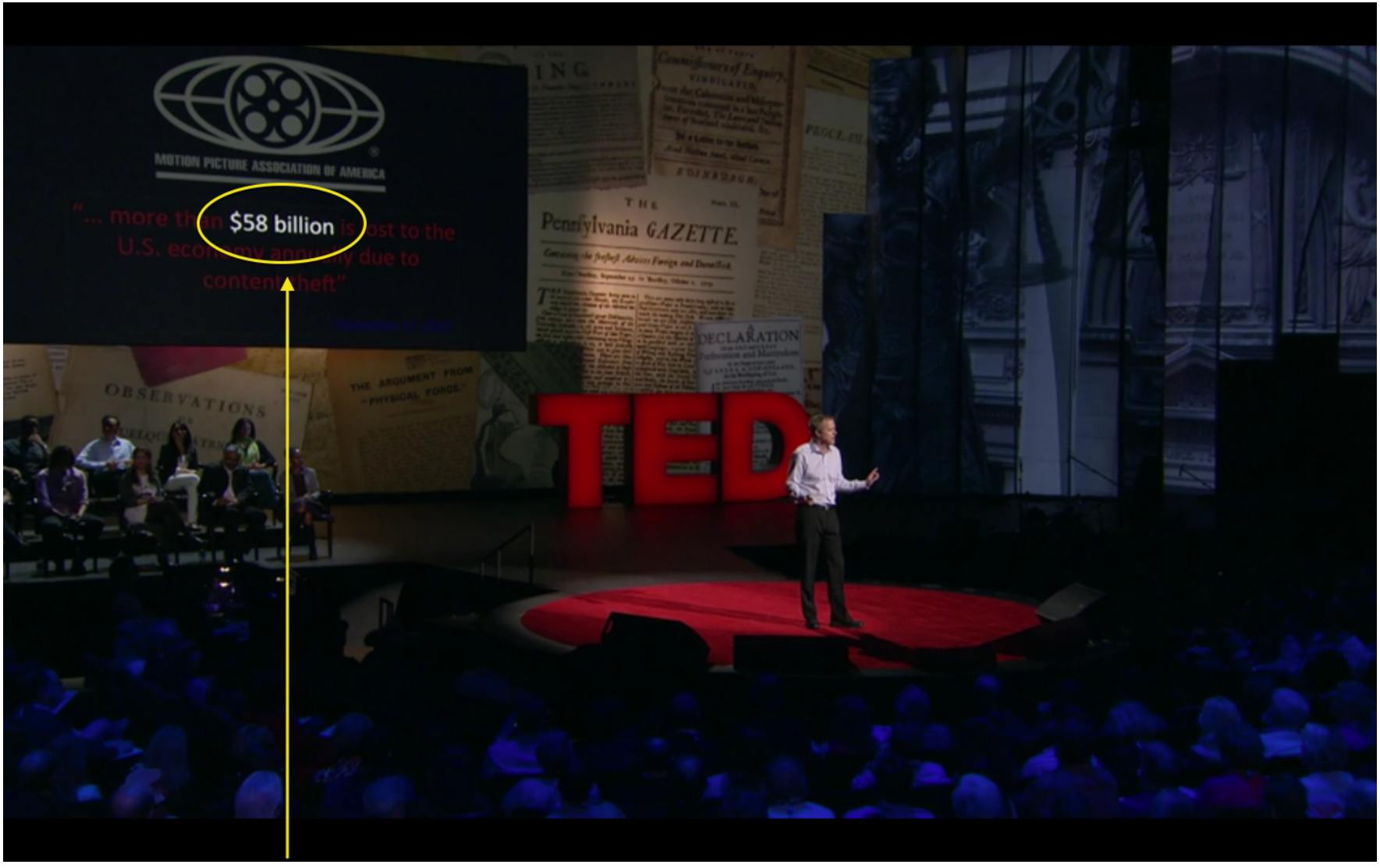
Reality expressed in **quantitative** terms

Quantitative and mathematical methods are a way to learn about complex phenomena by examining **formal relationships** between **numerical constructs** at several levels of application.



Our goal is to apply these methods to **reality** through **data**.

Reality is **measurable**



How much does your iPod cost?

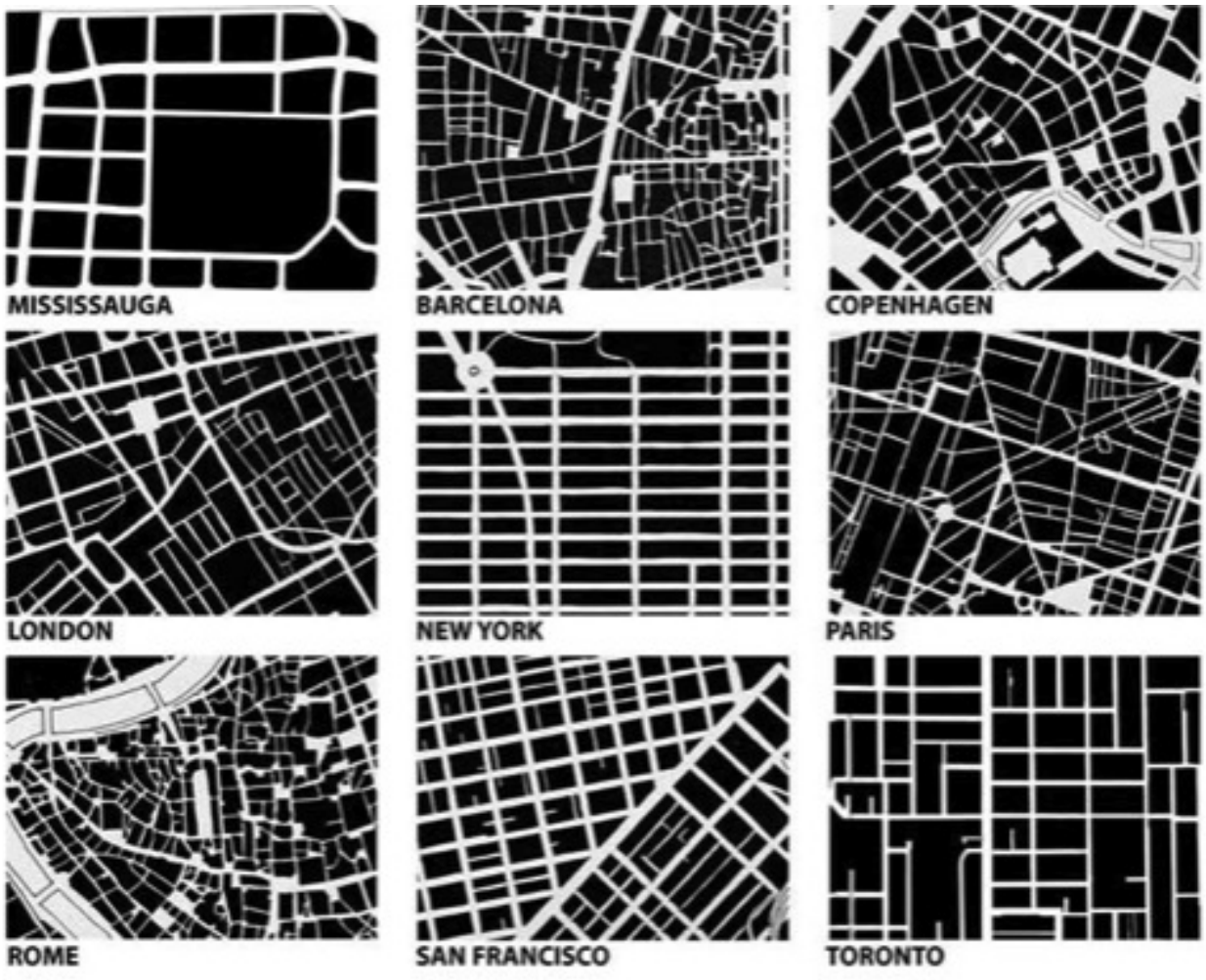
Reality is **predictable**

Los Angeles Times | ARTICLE COLLECTIONS

Stopping crime before it starts

Sophisticated analysis of data can sometimes tell police where criminals are headed. It's academic now, but the LAPD plans to get involved.

Reality is **visualizable**



Reality is **multidimensional**

Bad Religion (317 plays)

David Bowie (220 plays)

Isis (202 plays)

Horace Andy (178 plays)

Army of the Pharaohs (177 plays)

Biosphere (168 plays)

Bibio (162 plays)

Antonio Vivaldi (143 plays)

Neil Young (125 plays)

King Crimson (116 plays)

H.P. Lovecraft (116 plays)

Virgin Prunes (115 plays)

Motorama (109 plays)

Wax Tailor (104 plays)

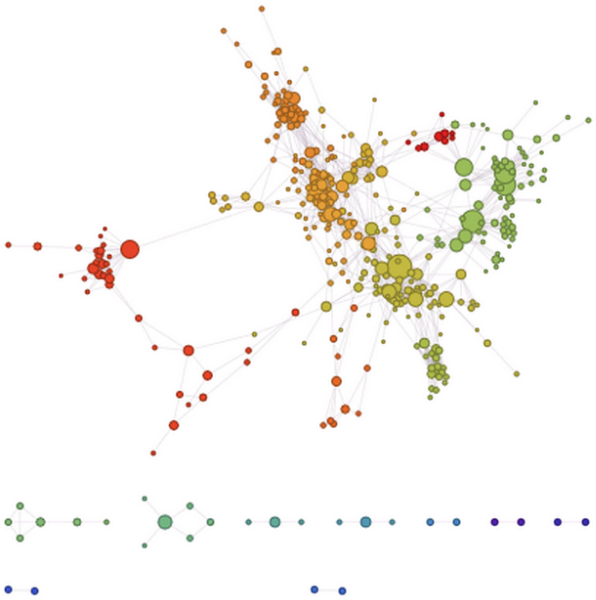
Lou Reed (103 plays)

Kate Bush (102 plays)

7L & Esoteric (102 plays)

Gonzales (100 plays)

Reality is **relational**



Friendship ties on Facebook

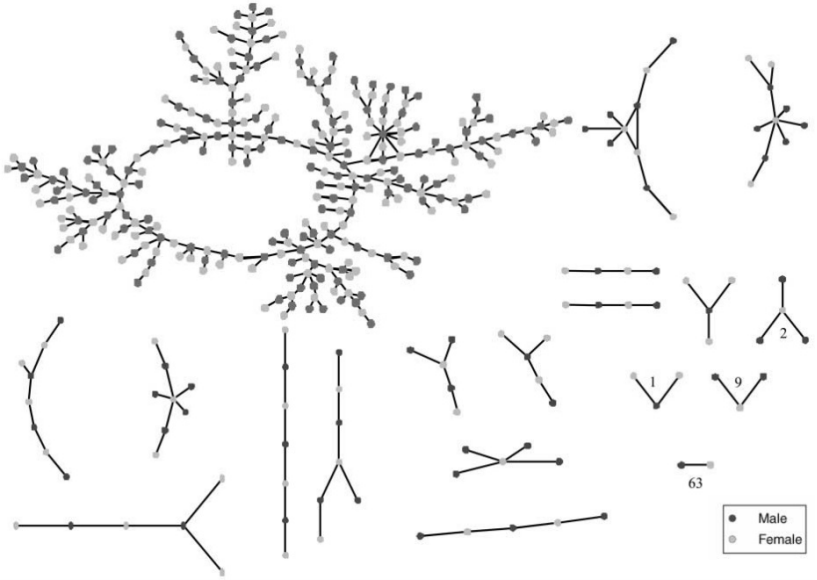


FIG. 2.—The direct relationship structure at Jefferson High

Sexual ties in high school

Data stand as **professional assets**

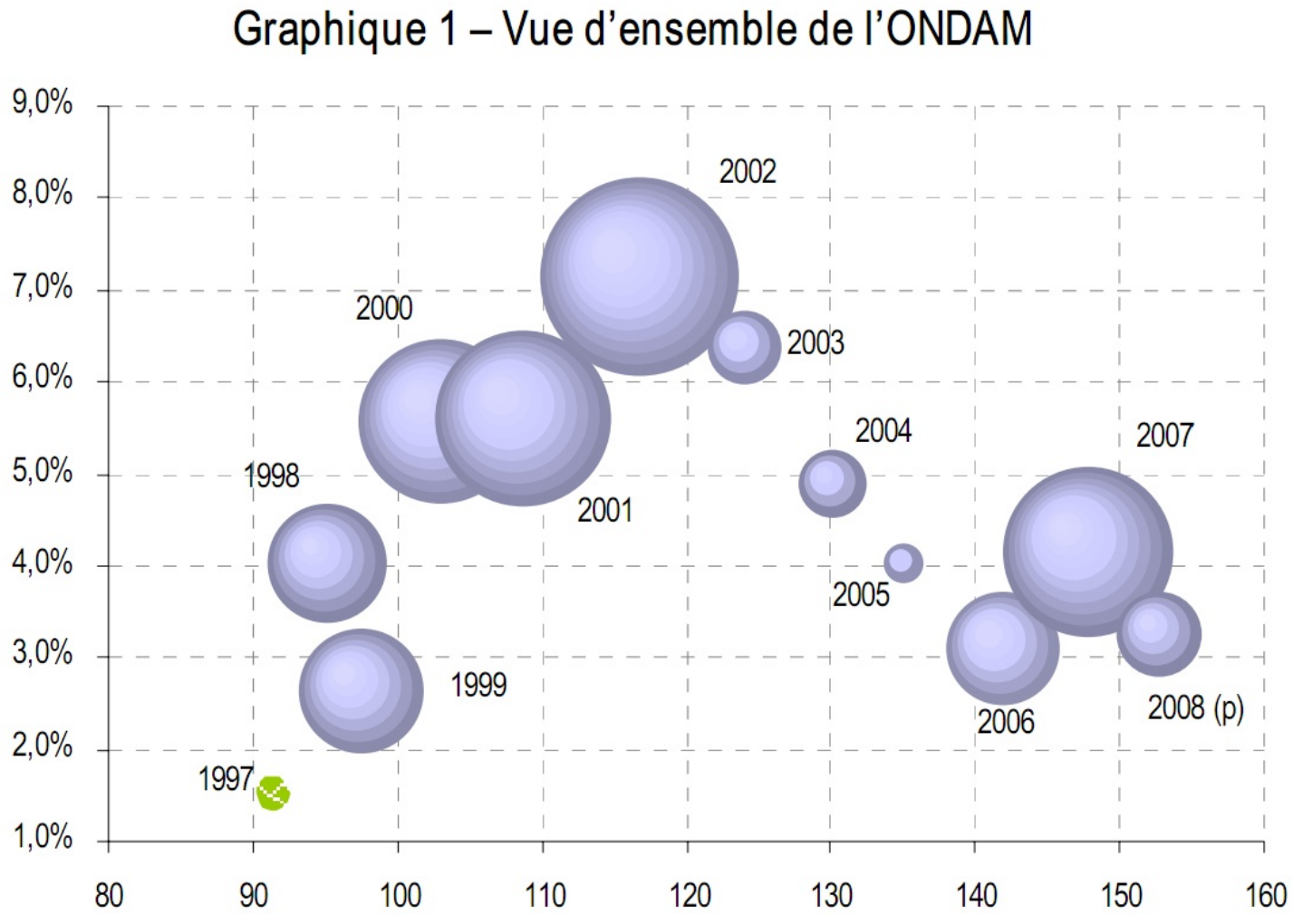
OECD Health Data 2010: Statistics and Indicators

AVAILABLE NOW - October 21st - Internet update for OECD Health Data 2010

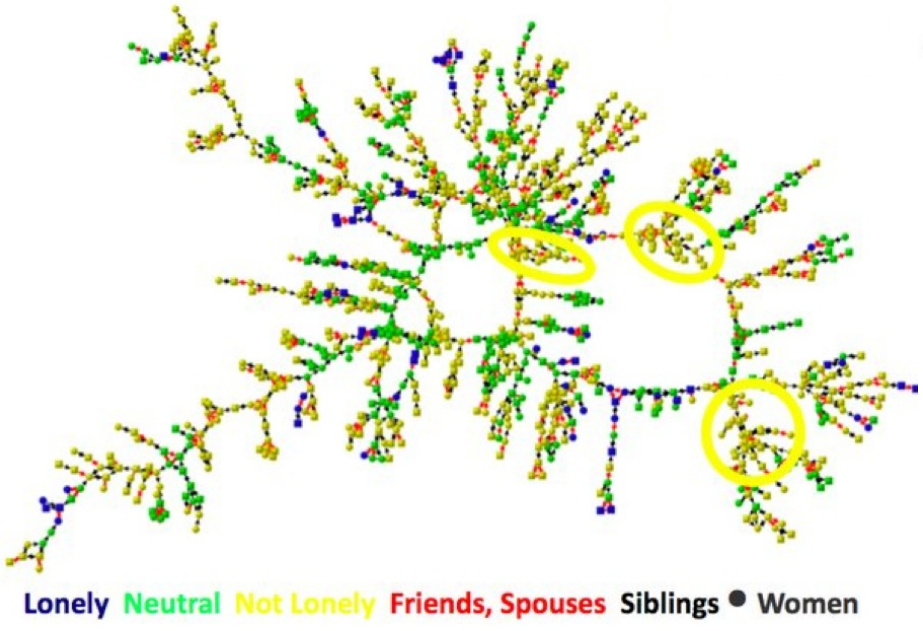
OECD Health Data 2010, released on 29 June 2010, offers the most comprehensive source of comparable statistics on health and health systems across OECD countries. It is an essential tool for health researchers and policy advisors in governments, the private sector and the academic community, to carry out comparative analyses and draw lessons from international comparisons of diverse health care systems.

- What is OECD Health Data 2010

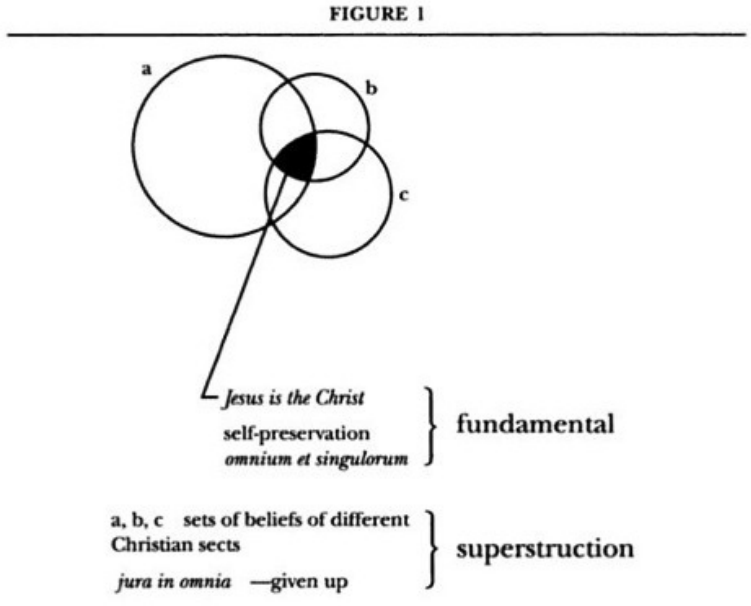
Data stand as **policy expertise**



Interpretation is key to all analysis



Loneliness in social networks



Sets of Christian beliefs

Interpretation is difficult

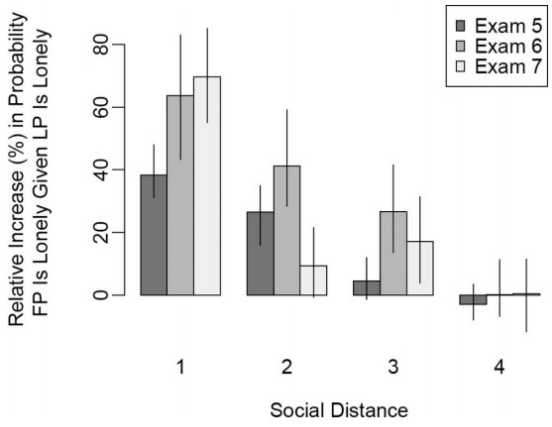
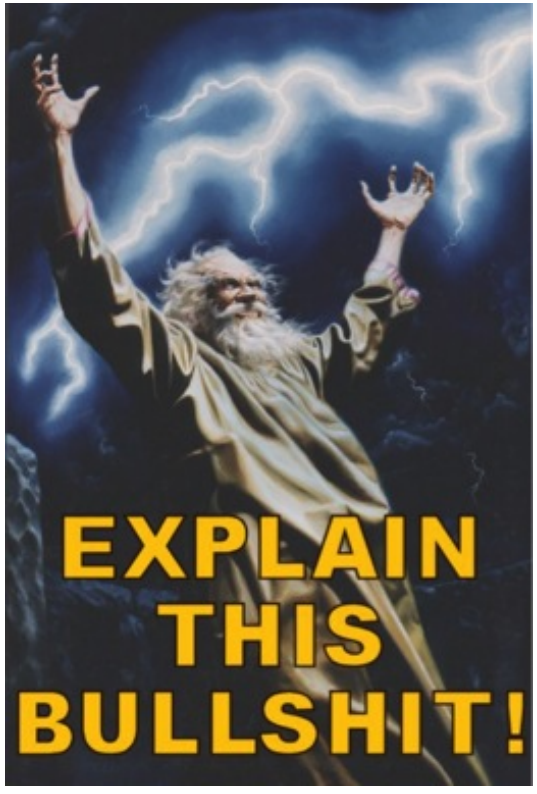


Figure 2. Social distance and loneliness in the Framingham Social Network. This figure shows for each exam the percentage increase in the likelihood a given focal participant (FP) is lonely if a friend or family member at a certain social distance is lonely (where lonely is defined as feeling lonely more than once a week). The relationship is strongest between individuals who are directly connected, but it remains significantly greater than zero at social distances up to three degrees of separation, meaning that a person's loneliness is associated with the loneliness of people up to three degrees removed from them in the network. Values are derived by comparing the conditional probability of being lonely in the observed network with an identical network (with topology and incidence of loneliness preserved) in which the same number of lonely participants are randomly distributed. Linked participant (LP) social distance refers to closest social distance between the LP and FP (LP = Distance 1, LP's LP = Distance 2, etc.). Error bars show 95% confidence intervals.

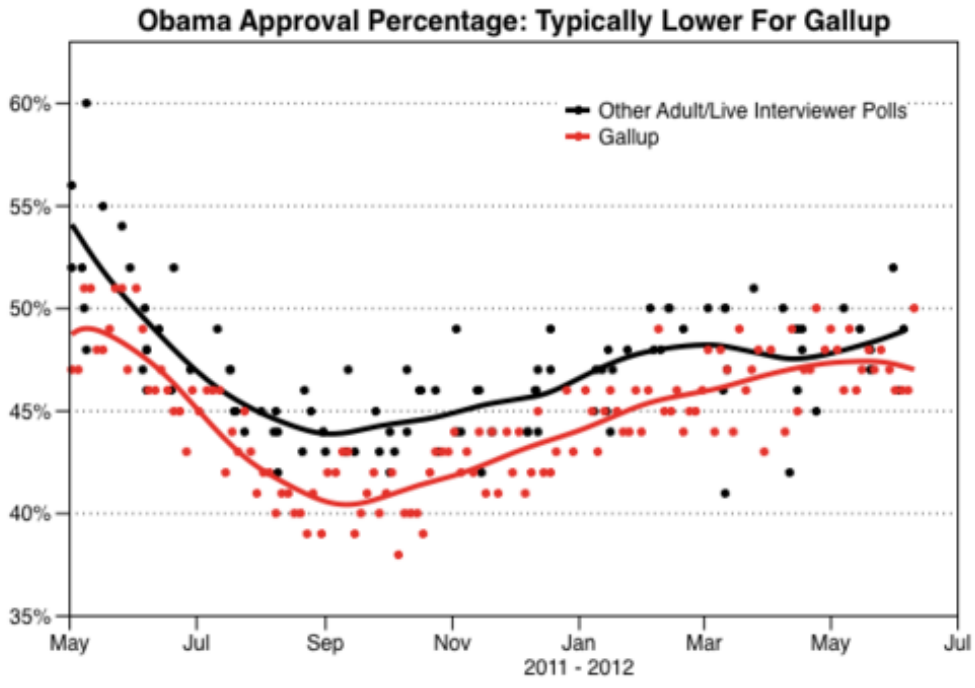
With explanation



Without explanation

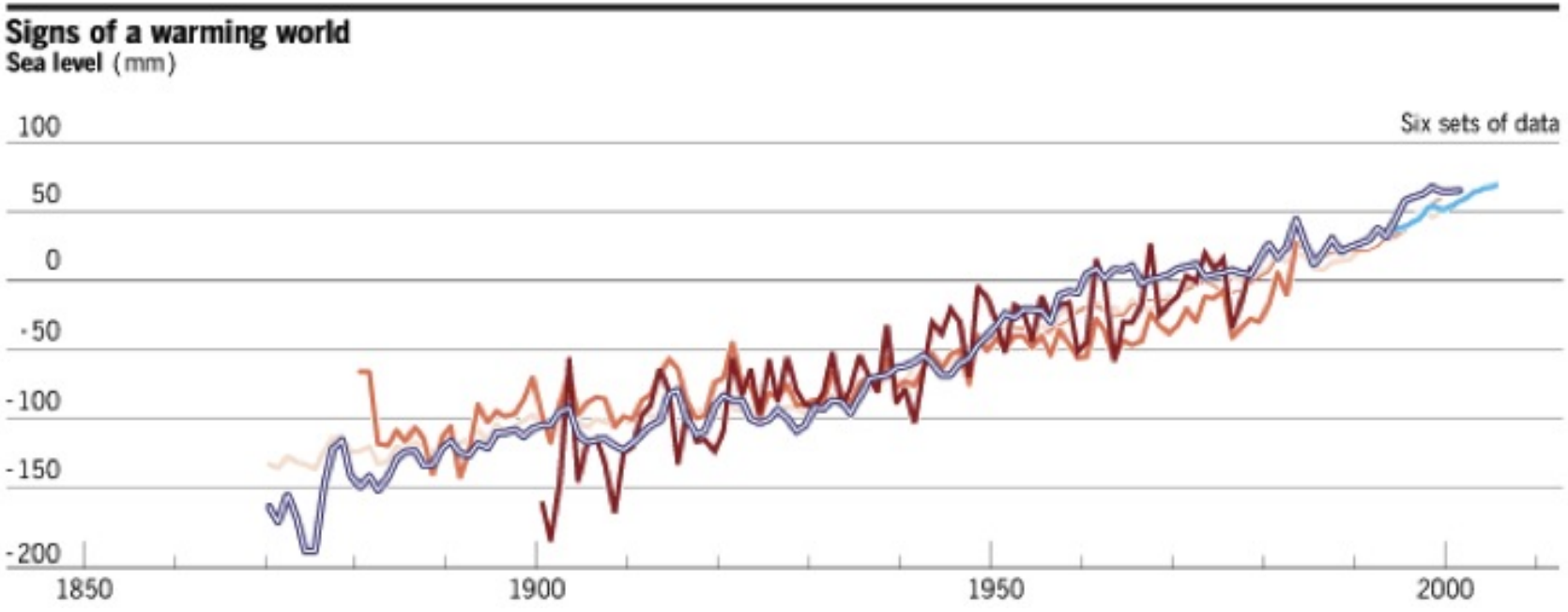
Internal biases in **surveys**

- Survey design
- Sample frame
- Question wording
- Measurements
- Cognitive limits
- Interference



External biases in **society**

- Media coverage
- Political spin
- Corporate obfuscation
- Lack of data



Further **WEIRD** biases

BEHAVIORAL AND BRAIN SCIENCES (2010), Page 1 of 75
doi:10.1017/S0140525X0999152X

The weirdest people in the world?

"The findings suggest that members of Western, Educated, Industrialized, Rich, and Democratic (WEIRD) societies, including young children, are among the least representative populations one could find for generalizing about humans."

Joseph Henrich
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http://www.psych.ubc.ca/~henrich/home.html

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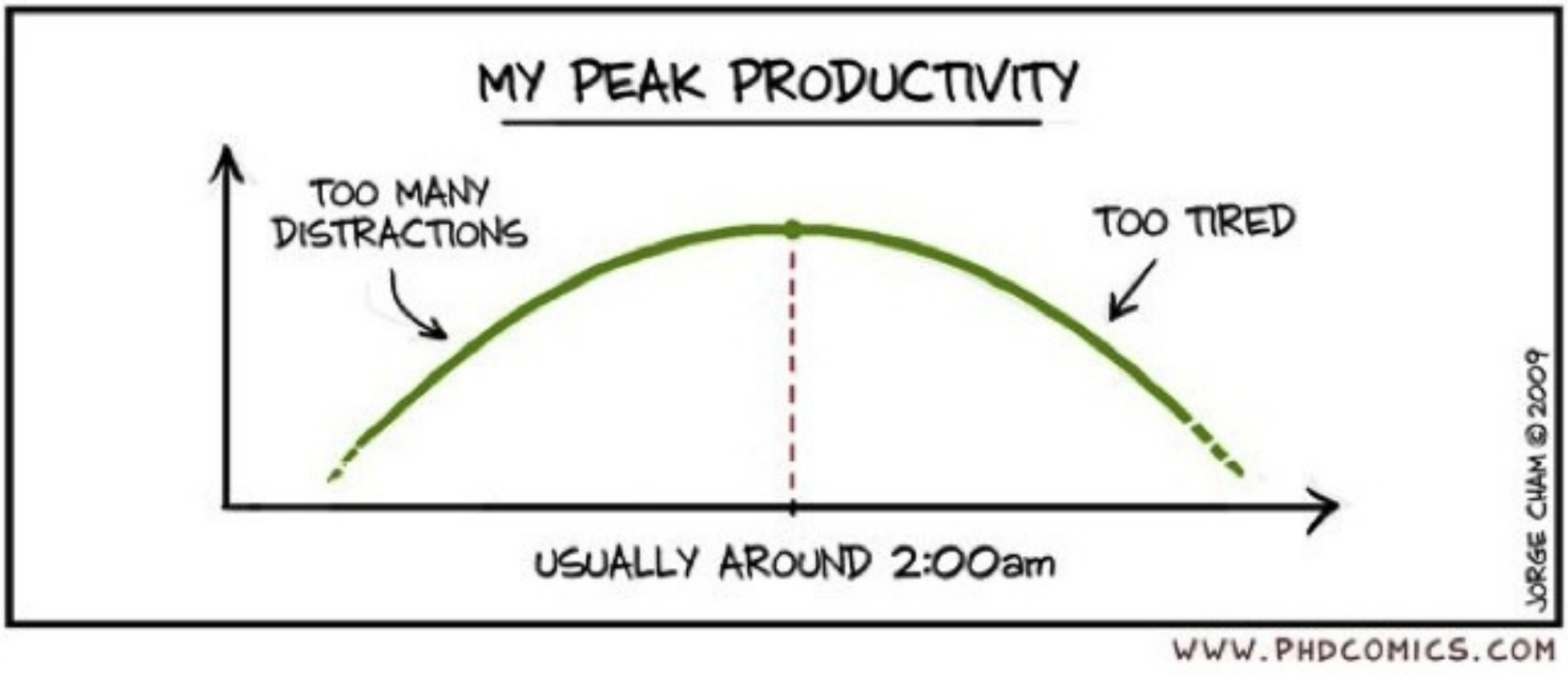
ara@psych.ubc.ca

Interpretation is what this course is eventually about



- What is the **measurement** of the axes?
- What is the **probability** of 2am being the "usual" peak point?
- What is the **shape** of the time/productivity relationship?

Course objectives

Course topics

- **Essential mathematics**: elementary calculus and probability
- **Frequentist statistics**: issues in measurement and estimation

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Course extensions

- **Economics**: Move to the Level 2 groups for a more advanced introduction with economic applications.
- **Programming**: An introduction to statistical computing will take place during the next semester.

Class rules

Attendance is compulsory.

Other internal regulations also apply to **grades, plagiarism** etc.
Please check these matters ASAP with admin.

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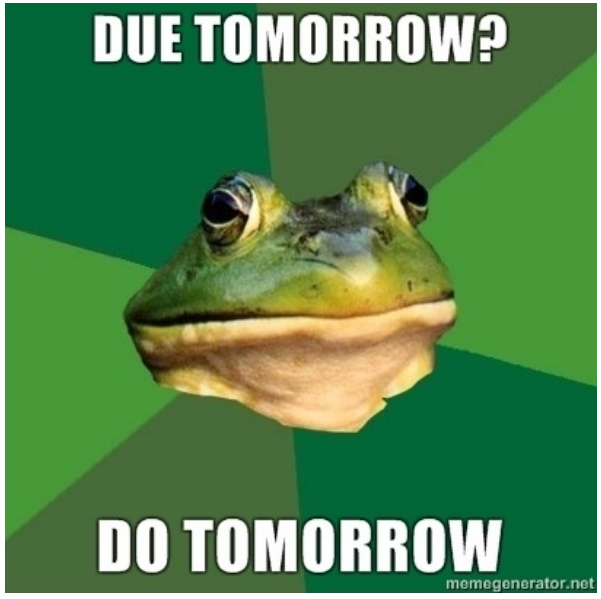
Regular work is an absolute requirement.

There will be plenty of course material to achieve all requirements and find help, but you will need to **read and practice weekly**.

Let me stress that:

Methods cannot be learned through overnight sessions or other techniques to catch up with late work. Sorry.



This won't work.

Class logistics

Elect a student representative!

No course without representation.

Any questions so far?

Do not worry about readings, homework, exams and grades: these will be discussed in class and details will reach you by email.

Calculus prefresher

Real numbers

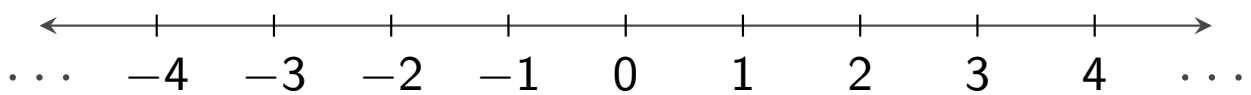
Real Numbers

Some things can be represented with numbers like $-1, 0, 8.697$ or 10.6 trillion (10,600,000,000,000).

All these numbers belong to the set of **real numbers** $\mathbb{R}$. Calculus is based on the real number system.

Integers

To explore real numbers, we start with **integers**, which can be either positive or negative:



Rational numbers

Rational numbers

We complete the set by adding ratios of integers, or fractions, a.k.a. **rational numbers**:

$r = \frac{m}{n}$ where m and n are integers and $n \neq 0$

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Irrational numbers

Some numbers like $\sqrt{2}$ are called irrational numbers because they cannot be expressed as a ratio of integers.
Irrational numbers nonetheless belong to the real number line.

Functions

Domain and range

A function f (or g, or h, or anything...) consists of two things:

- A set D of objects, called the *domain* of the function;
- A rule which associates to each object x in D another object, called the value of f at x. When abstract, this second object will be written $f(x)$.
- The set of all values of the function is called its **range**.

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Real-valued functions

- When the domain is the set $\mathbb{R}$ of real numbers, we call f a real-valued function.
- A real-valued function can be represented by its **curve**.

Functions

- Affine functions $f : x \to ax + b$

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- Power functions $g : y \to y^n$

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And of course, sums, products and compositions exist for all these functions : $1 + \sqrt{x}$, $\frac{t^2}{7+12t^4}$, etc.

We'll be using such notation to translate real-world problems into mathematical language.

Thank you for your attention

and welcome to the course!

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