Probability exam answers

Problem 1, Set 1

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$(\frac{1}{4})(.24) + (\frac{1}{2})(.09) - (\frac{1}{4})(.12) = .075$

Problem 1, Set 2: 11% and 10.5% (opposite conclusion).

Problem 2, Set 2, Question (a)

Assume a sample of 30 people randomly selected from a population where 10% failed to pay rent last month.

Calculate the mean and variance of failure to pay rent in the sample. Provide the formula for each.

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The random variable X, for which $P(X)$ designates failure to pay rent, follows a binomial distribution. Therefore:

$\mu = np = (30)(.1) = 3$

$\sigma^2 = np(1-p) = (30)(.1)(.9) = 2.7$

Problem 2, Set 1: $\mu = 5$, $\sigma = \sqrt{4} = 2$ (standard deviation).

Problem 2, Set 2, Question (b)

How likely is it to pick three people at random in this [random] sample and get only one who failed to pay rent? Provide the formula used.

For a single success $k = 1$ out of $n = 3$ independent trials, the probability of a random binomial variable X is $P(X = 1) = p(1 - p)^{n-1}$. Therefore:

$(.9) \cdot (.9) \cdot (.1) = (.9)^2(.1) = \frac{.81}{10} = .081$

Problem 2, Set 1: $P(X = 1) = (.8)^2(.2) = .128$.

Problem 3, Set 1, Question (a)

A country usually emits 8 tons of carbon dioxide CO_2 emissions per capita per year.

Annual trade sanctions can coerce this country into lowering its level of CO_2 emissions by $s = 25\%$. The probability of successful coercion through trade sanctions is observed to be $p = 20\%$.

How much CO_2 emissions is this country expected to produce on average?

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How much CO_2 emissions is this country expected to produce on average?

Absent of coercion, $x_0 = 8$. With coercion, $x_1 = 8 - (8 \cdot .25) = 6$.

$E(X) = \sum xP(x) = (.8)(8) + (.2)(6) = 6.4 + 1.2 = 7.6$

Problem 3, Set 2: $E(X) = (.8)(20) + (.2)(16) = 19.2$.

Problem 3, Set 1, Question (b)

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The maximal level of CO_2 emissions per capita-year is $P(x_0) = 8$.

The minimal level of CO_2 emissions per capita-year is $P(x_1) = 6$.

It is impossible for the random variable x to reach less than $P(x_1) = 6$ on average. The probability of sanctions would have to exceed 100% for $E(x)$ to fall below 6, which is inconceivable.

Problem 3, Set 2: same answer to Question (d).

Problem 3, Set 1, Question (c)

Find the level of sanction s at which that country emits less than 7 tons of CO_2 per capita on average.

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To solve $E(X) < 7$, let x be the level of emissions after trade sanctions, with $s = 1 - x$ the level of sanctions.

$(.2)(8x) + (.8)(8) < 7$

$\frac{8x}{5} < 7 - 6.4 = .6$ therefore $x < \frac{.6 \cdot 5 = 3}{8} = .375$

Sanctions need to be greater than $1 - x = .625$ (62.5%) for average emissions to drop at 7 or less.

Problem 3, Set 2, Question (b): $s = 1 - x > \frac{1}{2}$.

Problem 3, Set 1, Question (d)

Alternately, if the level of sanctions cannot be increased, how large should p be to attain that same maximum level of CO_2 emissions?

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Solving $E(x) < 7$ with $x = p$ the probability of sanctions:

$(x)(6) + (1 - x)(8) < 7$

$-2x < -1$ therefore $x > \frac{1}{2}$

If the sanctions applied more than 50% of years, the country would emit less than 7 tons per capita-year.

Problem 3, Set 2, Question (c): same answer.