QMM 6

Calculus exam answers

Problem 1, Set 1

Records indicate that x years after 2005, the average tax on a product is equal to $T(x) = 20x^2 + 40x + 600$ dollars.

At what rate was the tax increasing with respect to time in 2005?

$T'(0) = 40$

By how much did the tax change between the years 2005 and 2009?

$T(4) = 20(16) + 40(4) + 600 = 480 + 600$

$T(0) = 600 \quad T(4) - T(0) = 480$

Other answers

Set 2

$T(x) = x^3 + 40x^2 + 15x + 200 \quad T'(0) = 15$
$T(5) = 125 + 40(25) + 15(5) + 200 = 1200 + 200$
$T(0) = 200 \quad T(5) - T(0) = 1200$

Set 3

$T(x) = 10x^2 + 40x + 300 \quad T'(0) = 40$
$T(3) = 10(9) + 40(3) + 300 = 210 + 300$
$T(0) = 300 \quad T(5) - T(0) = 210$

Set 4

$T(x) = 20x^3 + 40x^2 + 15x + 800 \quad T'(0) = 15$
$T(2) = 20(8) + 40(4) + 15(2) + 800 = 350 + 800$
$T(0) = 800 \quad T(2) - T(0) = 350$

Problem 2, Set 1

An environmental study of an urban environment suggests that t years from now, the average level of carbon dioxide in the air will be $Q(t) = 0.15t^2 + 0.1t + 3.4$ parts per million.

At what rate will the carbon dioxide level be changing with respect to time 1 year from now?

$Q'(1) = 2(.15) + .1 = .4$

By how much will the carbon dioxide level change this year?

$Q(1) - Q(0) = (.15 + .1 + 3.4) - 3.4 = .25$

... over the next 2 years?

$Q(2) = .15(4) + .1(2) + 3.4 = .8 + 3.4 \quad Q(2) - Q(0) = .8$

Other answers

Set 2

$Q(t) = 0.05t^2 + 0.1t + 3.4 \quad Q'(1) = 2 * .05 + .1 = .2$
$Q(1) - Q(0) = (.05 + .1 + 3.4) - 3.4 = .15$
$Q(2) = .05(4) + .1(2) + 3.4 = .4 + 3.4 \quad Q(2) - Q(0) = .4$

Set 3

$Q(t) = -0.15t^2 - 0.1t + 3.4 \quad Q'(1) = -2(.15) - .1 = -.4$
$Q(1) - Q(0) = (-.15 - .1 + 3.4) - 3.4 = -.25$
$Q(2) = -.15(4) - .1(2) + 3.4 = -.8 + 3.4 \quad Q(2) - Q(0) = -.8$

Set 4

$Q(t) = 0.05t^2 - 0.1t + 3.4 \quad Q'(1) = 2 * .05 - .1 = 0$ (tricky)
$Q(1) - Q(0) = (.05 - .1 + 3.4) - 3.4 = -.05$
$Q(2) = .05(4) - .1(2) + 3.4 = 0 + 3.4 \quad Q(2) - Q(0) = 0$ (tricky)

Problem 3, Set 1

The population density at the centre of a city is 44,000 inhabitants. It then drops to 11,000 at a distance of 9 miles from the centre.

Express population as a function of the form $D(x) = Ae^{kx}$ where x is the distance in miles from the centre.

$A = D(0) = 44$

If $D(9) = 11$, then $11 = 44e^{-9k}$

$\frac{1}{4} = e^{-9k} \rightarrow \ln 1/4 = -9k \quad k = \frac{-\ln 1/4}{9}$

Note that k is negative when the function expresses exponential *decay*, as in Sets 1 and 3.

Other answers

Set 2

$A = D(0) = 7$

$D(3) = 21 \quad 21 = 7e^{3k} \quad 3 = e^{3k}$

$\ln 3 = 3k \quad k = \ln(3)/3$

Set 3

$A = D(0) = 24$

$D(5) = 4 \quad 4 = 24e^{-5k} \quad 1/6 = e^{-5k}$

$\ln 1/6 = -5k \quad k = \frac{-\ln 1/6}{5}$ (decay)

Set 4

$A = D(0) = 2$

$D(7) = 12 \quad 12 = 2e^{7k} \quad 6 = e^{7k}$

$\ln 6 = 7k \quad k = \frac{\ln 6}{7}$

Problem 4, Set 1

A country experiences a GDP decay equal to $G(t) = -3t^2 + 20t + 1800$ billion dollars in 2000.

What is its GDP decay rate in 2020?

$G'(t) = -6t + 20 \quad G'(20) = -6(20) + 20 = -100$

What is the relative decay rate of GDP in that same year?

$G(20) = -3(400) + 20(20) + 1800 = 1000$

$100\frac{G'(20)}{G(20)} = \frac{-100}{1000} = -.1$ -10% per year

Other answers

Set 2

$G(t) = -4t^2 + 100t + 300$

$G'(t) = -8t + 100 \quad G'(5) = -8(5) + 100 = 60$

$G(5) = -4(25) + 100(5) + 300 = 800 - 100 = 700 \quad \frac{60}{700} \approx 8.5\%$

Set 3

$G(t) = 3t^2 + 30t - 300$

$G'(t) = 6t + 30 \quad G'(20) = 6(20) + 30 = 150$

$G(20) = 3(400) + 600 - 300 = 1500 \quad \frac{150}{1500} = 10\%$

Set 4

$G(t) = 4t^2 - 5t + 25$

$G'(t) = 8t - 5 \quad G'(10) = 8(10) - 5 = 75$

$G(10) = 4(100) - 50 + 25 = 375 \quad \frac{75}{375} = 20\%$